Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2019**

**Subject Code: 2130002** **Date: 30/05/2019**

**Subject Name:Advanced Engineering Mathematics**

**Time: 02:30 PM TO 05:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | MARKS |
|---|---|---|---|
| **Q.1** | **(a)** | Solve $(x+y-2)\,dx+(x-y+4)\,dy=0$ | **03** |
| | **(b)** | Solve $(1+y^2)+(x-e^{-\tan^{-1}y})\dfrac{dx}{dy}=0$ | **04** |
| | **(c)** | Expand $f(x)=\lvert\cos x\rvert$ as a Fourier series in the interval $-\pi<x<\pi$ | **07** |
| **Q.2** | **(a)** | Define unit step function and unit impulse function. Also sketch the graphs. | **03** |
| | **(b)** | Solve $\left(\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}+y\right)=4\sin 2x$ | **04** |
| | **(c)** | Find the series solution of $y''+xy'+y=0$ about the ordinary point $x=0$. | **07** |
| | | **OR** | |
| | **(c)** | Find the Fourier series expansion for $f(x)$, if $f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$, Also deduce that $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\dots\dots=\dfrac{\pi^2}{8}$ | **07** |
| **Q.3** | **(a)** | Using Fourier integral representation, show that $\displaystyle\int_0^\infty \frac{1-\cos\pi\omega}{\omega}\sin\omega x\,d\omega=\begin{cases}\dfrac{\pi}{2}, & 0<x<\pi\\ 0, & x>\pi\end{cases}$ | **03** |
| | **(b)** | Solve $\left(\dfrac{d^2y}{dx^2}+y\right)=x^2\sin 2x$ | **04** |
| | **(c)** | Solve by method of variation of parameters $\left(\dfrac{d^2y}{dx^2}+9y\right)=\dfrac{1}{1+\sin 3x}$ | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | Find Laplace transform of $te^{at}\sin at$ | **03** |
| | **(b)** | Solve $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=5e^x-\sin 2x$ | **04** |

**(c)** Solve

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+4y=\cos(\log x)+x\sin(\log x)$$ **07**

**Q.4** **(a)** Find the orthogonal trajectories of the curve $y=x^2+c$ **03**

**(b)** Find the Laplace transform of (i) $\cos(at+b)$ **04**

(ii) $\sin^2 3t$

**(c)** State convolution theorem and apply it to evaluate $L^{-1}\left(\frac{s^2}{\left(s^2+4\right)^2}\right)$ **07**

**OR**

**Q.4**

**(a)** Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4y=0$ **03**

**(b)** Find Half range cosine series for $f(x)=(x-1)^2$ in the interval $0<x<1$ **04**

**(c)** Solve $y''+4y'+3y=e^{-t}, y(0)=y'(0)=1$ using Laplace transform. **07**

**Q.5** **(a)** Form the partial differential equation by eliminating the arbitrary constants from $z=ax+by+a^2+b^2$ **03**

**(b)** Solve $(y-z)p+(x-y)q=z-x$ **04**

**(c)** Solve $3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$, where $u(x,0)=4e^{-x}$ using the method of separation of variables. **07**

**OR**

**Q.5** **(a)** Form the partial differential equation by eliminating the arbitrary function from $f(x^2+y^2, z-xy)=0$ **03**

**(b)** Solve $\log\left(\frac{\partial^2 z}{\partial x\partial y}\right)=x+y.$ **04**

**(c)** A bar with insulated sides is initially at temperature $0^0C$, throughout. The end $x=0$ is kept at $0^0C$ and heat is suddenly applied at the end $x=l$ so that $\frac{\partial u}{\partial x}=A$ for $x=l$, where A is a constant. Find the temperature function. **07**

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